

① Find $\frac{d}{dx} \left(\pm \sqrt{\frac{8-x^4}{2}} \right)$

② Give the chain rule formula

③ What does $\frac{dy}{dx}$ mean? Be as thorough as possible.

$$\pm \sqrt{\frac{8-x^4}{2}} \quad \pm \sqrt{4-\frac{1}{2}x^4} \quad y = \pm \sqrt{u} \quad u = 4 - \frac{1}{2}x^4$$

$$\frac{dy}{du} = \frac{1}{2\sqrt{u}} \quad \frac{du}{dx} = -2x^3 \quad \frac{dy}{dx} = \pm \frac{2x^3}{2\sqrt{4-\frac{1}{2}x^4}} = \pm \frac{x^3}{\sqrt{4-\frac{1}{2}x^4}}$$

$$x^4 + 2y^2 = 8$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

↓

change in y in relation to change in x

derivative of y with respect to x

instantaneous rate of change

Implicit Differentiation

- easier in some cases

- allows to find more derivatives

Explicit

$$y^2 = x$$

$$y = \pm \sqrt{x}$$

$$\frac{dy}{dx} = \pm \frac{1}{2\sqrt{x}}$$

Implicit

$$y^2 = x$$

$$2y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2y}$$

$$x^4 + 2y^2 = 8$$

$$4x^3 \frac{dx}{dx}$$

$$4x^3 + 4y' \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{4x^3}{4y} = \left(-\frac{x^3}{y} \right)$$

$$2y = x^2 + \sin y$$

$$2 \frac{dy}{dx} = 2x + \cos(y) \frac{dy}{dx} - \cos(y) \frac{dy}{dx}$$

$$2 \frac{dy}{dx} - \cos(y) \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} (2 - \cos(y)) = 2x$$

$$\frac{dy}{dx} = \frac{2x}{2 - \cos y}$$

Try

$$x^2 - xy + y^2 = 7$$

$$2x - \left(x \frac{dy}{dx} + y(1) \right) + 2y \frac{dy}{dx} = 0$$

$$\underline{2x} - x \frac{dy}{dx} - \underline{y} + 2y \frac{dy}{dx} = 0$$

$$-x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2x + y$$

$$\frac{dy}{dx} (-x + 2y) = -2x + y$$

$$\frac{dy}{dx} = \frac{-2x + y}{-x + 2y}$$

$x^2 + y^2 = 25$, find slope at the point $(3, -4)$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y} \Big|_{(3, -4)} = -\frac{3}{-4} = \left(\frac{3}{4}\right)$$

$$\frac{d^2 y}{dx^2} \text{ of } 2x^3 - 3y^2 = 8$$

$$6x^2 - 6y \frac{dy}{dx} = 0$$

$$6(x^2 - y \frac{dy}{dx}) = 0$$

$$x^2 - y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^2}{-y} = \left(\frac{x^2}{y} \right)$$

$$\frac{d^2 y}{dx^2} = \frac{y(2x) - x^2 \frac{dy}{dx}}{y^2} = \frac{2xy - \frac{x^4}{y}}{y^2}$$

Sect. 3.7

1, 4, 5, 6, 7, 10, 11, 13, 16, ~~18, 19, 22, 23, 25, 29, 30~~